om a

string

ual to

m_2.

Art-10. Two masses m_1, m_2 are connected by an inelastic string ; m_2 is placed on a smooth horizontal table and the string passes over a light smooth pulley at the edge of the table and m_1 is hanging freely. Determine the motion and the tension in the string. Find also the pressure on the pulley.

(P.U. 2013, 2014)

Proof : Let l be the length of the string and at any instant let x and y be the distances of masses m_2 and m_1 respectively from the edge of the table, then

$$x + y = l$$

$$\therefore \quad \frac{dx}{dt} + \frac{dy}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{dy}{dt} \text{ and } \frac{d^2x}{dt^2} = -\frac{d^2y}{dt^2}$$

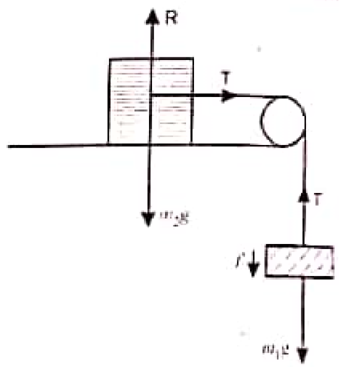
Thus the velocity and acceleration of m_1, vertically downwards are at all time equal to those of m_2 along the table towards the edge. Let f be the common acceleration and T be the tension of the string.

The equation of motion for mass m_1 is

$$m_1 f = m_1 g - T \quad \ldots(1)$$

The equation of motion for mass m_2 in a horizontal direction is

$$m_2 f = T \quad \ldots(2)$$



Adding (1) and (2), we get,

$$(m_1 + m_2) f = m_1 g$$

$$\therefore \quad f = \frac{m_1}{m_1 + m_2} g$$

Putting this value of f in (2), we get,

$$T = \frac{m_1 m_2}{m_1 + m_2} g$$

Let R be the reaction of the table on the mass m_2. Since m_2 does not move in a vertical direction,

$$\therefore \quad m_2 . 0 = R - m_2 g \Rightarrow R = m_2 g$$

The pulley is being pressed by the two equal forces T at right angles

$$\therefore \quad \text{pressure on the pulley} = \sqrt{T^2 + T^2} = \sqrt{2T^2} = \sqrt{2}\, T = \sqrt{2}\, \frac{m_1 m_2}{m_1 + m_2} g$$

Note : If the table is rough instead of smooth, the force of friction of amount μR acts on the mass m_2 in the direction of the string opposite to that of the tension, so that equation (2) becomes

$$m_2 f = T - \mu R \quad i.e., \quad m_2 f = T - \mu m_2 g$$

Solving this equation with $m_1 f = m_1 g - T$, we get,

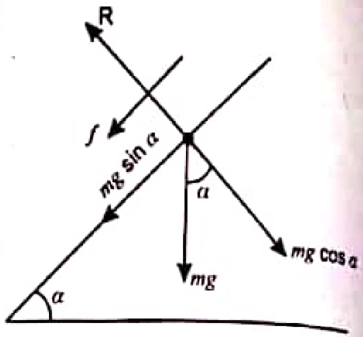
$$f = \frac{m_1 - \mu m_2}{m_1 + m_2} g, \quad T = \frac{m_1 m_2 (1 + \mu)}{m_1 + m_2}$$

The body moves only when $m_1 > \mu m_2$.

Art-11. Motion on a Smooth Inclined Plane

A body moves down a smooth inclined plane under the action of gravity alone, discuss its motion.

Proof : Let m be the mass of the body and α the inclination of the plane.



Since the plane is smooth, therefore there is no force of friction parallel to the plane opposing the motion of the mass.

The only forces acting on the body are :

(*i*) its weight mg acting vertically downwards

(*ii*) the reaction R acting perpendicular to the plane.

The vertical force mg is equivalent to :

(*i*) a force $mg \cos \alpha$ perpendicular to the plane and

(*ii*) a force $mg \sin \alpha$ down the plane.

Since the mass m moves down the plane, its acceleration f acts down the plane.

Since there is no motion and, therefore, no acceleration perpendicular to the plane.

$\therefore \quad R - m\,g \cos \alpha = m \times 0$

$\Rightarrow \quad R = m\,g \cos \alpha$ ...(1)

The motion of the body along the plane is given by

$m\,f = m\,g \sin \alpha \quad \Rightarrow \quad f = g \sin \alpha$ ...(2)

Hence the body moves down the plane with a constant acceleration $g \sin \alpha$ and it is, therefore, evident that the motion of a body, on a smooth inclined plane is similar to that of a body moving vertically under gravity, except that instead of g we have to take $g \sin \alpha$ for acceleration.

Thus, if a body is projected up a smooth inclined plane along a line of greatest slope with an initial velocity u, then v, the velocity, and s, the distance described in time t are given by the equations.

$$v = u - g \sin \alpha \,.\, t,$$

$$s = u\,t - \frac{1}{2} g \sin \alpha \,.\, t^2$$

and $$v^2 = u^2 - 2\,g \sin \alpha \,.\, s$$

Cor. 1. Time to reach the highest point

At the highest point, $v = 0$

$$\therefore \quad 0 = u - g \sin \alpha \,.\, t \quad \Rightarrow \quad t = \frac{u}{g \sin \alpha}$$

gravity alone.

mg cos α

Cor. 2. Distance of the highest point from the point of projection

At the highest point $v = 0$

$$\therefore \quad 0 = u^2 - 2\, g \sin\alpha \,.\, s \quad \Rightarrow \quad s = \frac{u^2}{2\, g \sin\alpha}$$

Cor. 3. Time of Flight

Let T be the time taken by the body to return to the point of projection, so its displacement is zero.

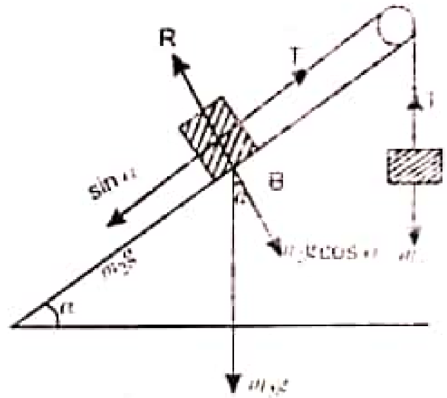
$$\therefore \quad 0 = u\,T - \frac{1}{2} g \sin\alpha \,.\, T^2,\ T \neq 0 \quad \Rightarrow \quad T = \frac{2u}{g \sin\alpha}$$

Note : The line of greatest slope through any point on an inclined plane is the line on the plane draws through that point perpendicular to the line of intersection of the inclined plane with a horizontal plane.

Art-12. Constrained Motion Along a Smooth Inclined Plane

A mass m_1 hanging vertically is connected to another mass m_2 placed on a smooth inclined plane of inclination α by means of a light inelastic string passing over a smooth pulley fixed at the top of the plane. The system is released from rest, discuss the motion.

Proof : Let mass m_1 move downwards with acceleration f.



Since m_2 is connected by in-extensible string passing over a smooth pulley, so mass m_2 will move up the plane with the same acceleration f and tension T, say, will be same throughout the string. Forces acting on mass m_2 are

(*i*) its weight $m_2 g$ acting vertically downwards which has got components $m_2 g \sin\alpha$ down the plane and $m_2 g \cos\alpha$ perpendicular to the plane.

(*ii*) normal reaction R and

(*iii*) tension T up the plane

Since m_2 has no motion at right angles to the plane

$\therefore \quad m_2 \times 0 = R - m_2 g \cos\alpha \Rightarrow R = m_2 g \cos\alpha$ ...(1)

Forces acting on mass m_1 are

(*i*) its weight $m_1 g$ acting vertically downwards

(*ii*) tension T acting vertically upwards.

The equation of motion of m_1 is $m_1 f = m_1 g - T$ ...(2)

The equation of motion of m_2 is $m_2 f = T - m_2 g \sin\alpha$ ...(3)

Adding (2) and (3), we get,

$$(m_1 + m_2) f = (m_1 - m_2 \sin\alpha) g$$

$$\therefore \quad f = \frac{m_1 - m_2 \sin\alpha}{m_1 + m_2} g \qquad ...(4)$$

From (2),

$$T = m_1 (g - f) = m_1 \left[g - \frac{m_1 - m_2 \sin\alpha}{m_1 + m_2} g \right] \qquad [\because \text{ of } (4)]$$

$$= m_1 g \left[\frac{m_1 + m_2 - m_1 + m_2 \sin\alpha}{m_1 + m_2}\right] = \frac{m_1 m_2 (1 + \sin\alpha)}{m_1 + m_2} g$$

Pressure on the pulley = resultant of two forces T, T at an angle $\left(\frac{\pi}{2} - \alpha\right)$.

$$= \sqrt{T^2 + T^2 + 2T.T\cos\left(\frac{\pi}{2} - \alpha\right)} = \sqrt{2T^2 + 2T^2 \sin\alpha} = \sqrt{2}\, T \sqrt{1 + \sin\alpha}$$

$$= \sqrt{2} \frac{m_1 m_2 (1 + \sin\alpha)^{\frac{3}{2}}}{m_1 + m_2} . g$$

Cor. 1. Condition for no motion

The system was initially at rest. It will have no motion if $f = 0$

i.e. if $\frac{m_1 - m_2 \sin\alpha}{m_1 + m_2} g = 0$

i.e. if $m_1 - m_2 \sin\alpha = 0$

i.e. if $\frac{m_1}{m_2} = \sin\alpha$

Hence the system has no motion if $\frac{m_1}{m_2} = \sin\alpha$.

Cor. 2. Condition for downward motion of m_1.

Mass m_1 will move down if $f > 0$

i.e. if $\frac{m_1 - m_2 \sin\alpha}{m_1 + m_2} g > 0$ *i.e.* if $m_1 - m_2 \sin\alpha > 0$

i.e. if $\frac{m_1}{m_2} > \sin\alpha$

Cor. 3. Condition for upward motion of m_1.

Mass m_1 will have move up if $f < 0$

i.e. if $\frac{m_1 - m_2 \sin\alpha}{m_1 + m_2} g < 0$ *i.e.* if $m_1 - m_2 \sin\alpha < 0$

i.e. if $\frac{m_1}{m_2} < \sin\alpha$

Note 1. If $\alpha = 0$, then $f = \dfrac{m_1}{m_1 + m_2} g$, $T = \dfrac{m_1 m_2}{m_1 + m_2} g$

which give acceleration and tension when m_2 is placed on a smooth horizontal table and m_1 hangs vertically.

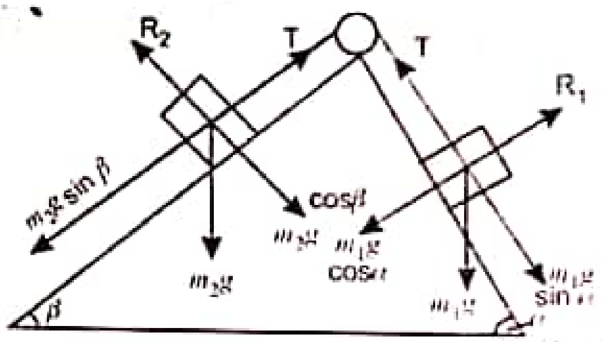
Note 2. If $\alpha = \dfrac{\pi}{2}$, then $f = \dfrac{m_1 - m_2}{m_1 + m_2} g$, $T = \dfrac{2 m_1 m_2}{m_1 + m_2} g$.

which give acceleration and tension in Atwood's machine.

Art-13. Two smooth inclined planes of equal heights and inclinations α and β are placed back to back. Masses m_1, m_2 resting on them are connected by a light inextensible string passing over a smooth pulley fixed at the common vertex of the two planes. If the system is free to move, discuss the motion.

Proof. Assume that

$\alpha > \beta$ and $m_1 > m_2$.



$\therefore$ the mass m_1 will descend the plane and mass m_2 will ascend the plane.

Let T be the tension and f be the acceleration.

The equation of motion of m_1 is

$$m_1 g \sin \alpha - T = m_1 f \quad \text{...(1)}$$

The equation of motion of m_2 is

$$T - m_2 g \sin \beta = m_2 f \quad \text{...(2)}$$

Adding (1) and (2), we get,

$$(m_1 \sin \alpha - m_2 \sin \beta) g = (m_1 + m_2) f$$

$$\Rightarrow \quad f = \frac{m_1 \sin \alpha - m_2 \sin \beta}{m_1 + m_2} \cdot g \quad \text{...(3)}$$

which gives the acceleration of the system.

From (1),

$$T = m_1 (g \sin \alpha - f)$$

$$= m_1 \left[g \sin \alpha - \frac{m_1 \sin \alpha - m_2 \sin \beta}{m_1 + m_2} \right] \quad [\because \text{ of } (3)]$$

$$= m_1 g \left[\frac{m_1 \sin\alpha + m_2 \sin\alpha - m_1 \sin\alpha + m_2 \sin\beta}{m_1 + m_2} \right]$$

$$= \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1 + m_2} g$$

which gives the tension.

Note. If $m_1 \sin\alpha < m_2 \sin\beta$, then the motion will be in the opposite direction.

If $m_1 \sin\alpha = m_2 \sin\beta$, then $f = 0$ and so the system will be either at rest or once if the system is put in motion with a certain velocity, it will continue to move with same velocity in the same direction.

Art-14. Newton's Law of Gravitational Attraction

Statement. Newton's law gravitational attraction states that the force of the attraction between two bodies varies directly to the product of their masses and inversely as the square of the distance between them.

The law is expressed by the equation

$$F = G\,\frac{M_1 M_2}{r^2}$$

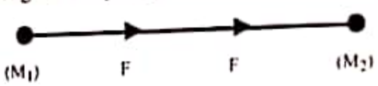
where F = the magnitude of the force of mutual attraction between the two bodies.

M_1, M_2 = the masses of the two bodies

r = the distance between the bodies

G = a universal constant known as the constant of gravitation which does not depend upon the nature of bodies involved.

The mutual forces F obey the law of action and reaction since they are equal and opposite and are directed along the line joining the centres of gravity of the two bodies.



Experimentally, the value of $G = 6{\cdot}67 \times 10^{-8}$ cm^3 (gm-sec^2)

$= 6{\cdot}67 \times 10^{-11}$ M.K.S. Units.

Gravitational forces exist between every pair of bodies.

The gravitational force of attraction F exerted by Earth on a particle of mass m is given by

$$F = \frac{GM\,m}{r^2} \qquad ...(1)$$

where M is the mass of Earth and r, the distance of the particle from the centre of the Earth.

If $m = 1$, the force F is called the acceleration due to gravity of the Earth and usually denoted by g. Thus

$$g = \frac{GM}{r^2} \qquad ...(2)$$

If g(0) denotes the value of g on the surface of Earth, then its value $g(h)$ at a height h above the surface is given by

$$g(h) = \frac{GM}{(r+h)^2} = \frac{GM}{r^2\left(1+\frac{h}{r}\right)^2} = \frac{GM}{r^2}\left(1+\frac{h}{r}\right)^{-2}$$

$$= \frac{GM}{r^2}\left(1 - \frac{2h}{r}\right), \text{ neglecting } \left(\frac{h}{r}\right)^2 \text{ and higher powers}$$

$$= g(0)\left(1 - \frac{2h}{r}\right) \quad \text{...(3)}$$

From (3), it is clear that $g(h)$ decreases as h increases if $\frac{h}{r} < 1$.

The weight of a body is usually denoted by W.

Using (1) and (2), $W = m\,g$ ...(4)

It is clear from (3) that the value of the weight $m\,g$ of a body decreases as the body is taken higher up the surface of Earth.

ILLUSTRATIVE EXAMPLES