# 4

# MOTION UNDER VARIABLE ACCELERATION

## Art-1. Introduction

In the last chapter, we got the fundamental equation of dynamics namely $P = mf$ where $m$ denotes the mass of the moving particle and P, the force acting on it at any instant along the line of its motion and measured in absolute units. Here $f$ denotes the acceleration Expressions for acceleration $f$ are $\frac{d^2s}{dt^2}$, $\frac{dv}{dt}$ or $v\frac{dv}{ds}$ where $s$ is the distance measured along the line from a fixed point on it. The equation $P = mf$ can be written as

$$m\frac{d^2s}{dt^2} = P \quad \ldots(1)$$

or $$m\frac{dv}{dt} = P \quad \ldots(2)$$

or $$mv\frac{dv}{ds} = P \quad \ldots(3)$$

When the force P or the acceleration $\frac{P}{m}$ is constant, any of these equations can be used. However if

(*i*) F is a function of $t$, (1) or (2) is used

(*ii*) F is a function of $v$, (2) or (3) is used

(*iii*) F is a function of $s$, only (3) is used.

The following examples illustrate the above cases :

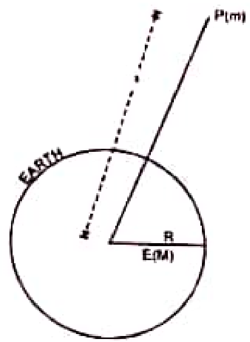
## ILLUSTRATIVE EXAMPLES

**Example 1.** A particle starts from rest and moves along a straight line with an acceleration $f$ varying as $t^n$. If $v$ be the velocity at a distance $s$ from the starting point, show that $(n+1)\,v^2 = (n+2)\,f s$.

(G.N.D.U. 2009 ; P.U. 2013)

## Art-2. Motion Under Gravity Outside the Surface of Earth

We have studied Newton's law of gravitational attraction which states : *If $M_1$ and $M_2$ are the masses of any two bodies in the universe, then the mutual attraction between them is given by $F = G\frac{M_1 M_2}{r^2}$, where r is the distance between the bodies and G is the universal constant of gravitation.*




It is this law which governs the motion of a body outside the surface of earth.

If M be the mass of the earth and $m$, that of the particle situated at a distance $x$ from the centre of earth, then F the force of attraction between them is given by

$$F = G\frac{Mm}{x^2} \quad \text{...(1)}$$

where G is the universal constant of gravitation. Due to this attraction, if $f$ is the acceleration generated in the particle, then

$$F = mf \quad \text{...(2)}$$

From (1) and (2), we get,

$$G\frac{Mm}{x^2} = mf$$

$$\Rightarrow \quad f = \frac{GM}{x^2} = \frac{k}{x^2} \quad \text{...(3)}$$

where $k = GM$ is constant

$$\Rightarrow \quad f \propto \frac{1}{x^2}$$

*Hence the acceleration produced in bodies outside the surface of earth is inversely proportional to the square of the distance of the body from the centre of the earth.*

On the surface of the earth, $x = R$, the radius of earth.

$$\therefore \quad f = \frac{GM}{R^2} = \frac{k}{R^2}$$

This value of $f$ is denoted by g, the acceleration due to gravity.

Thus $g = \frac{k}{R^2} \quad \Rightarrow \quad k = gR^2$

$\therefore$ from (3), $f = \frac{gR^2}{x^2}$

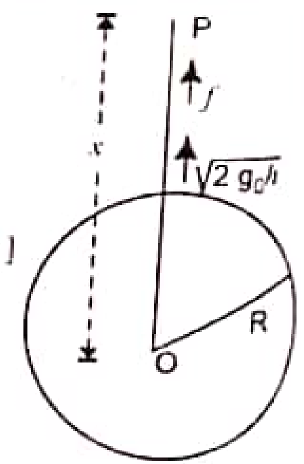
Hence the acceleration produced by the earth in a body situated at a distance $x$ from the centre of the earth is $\frac{g R^2}{x^2}$, directed towards the centre of earth, where R is the radius of earth and $g$ is acceleration due to gravity.

# ILLUSTRATIVE EXAMPLES

**Example 1.** A particle is projected upwards with velocity $\sqrt{2 g_0 h}$, where $g_0$ is the acceleration due to gravity on the surface of earth. Show that if the variation of gravity with height is taken into consideration then the particle will reach a height $H$ given by $\frac{1}{h} - \frac{1}{H} = \frac{1}{R}$ R being the radius of the earth.

**Sol.** Let P be the position of the particle at a distance $x$ (> R) from the centre of earth.

If $f$ is the acceleration at P along OP, then




$$f = -\frac{g_0 R^2}{x^2}$$

[$\because$ acceleration along PO $= \frac{g_0 R^2}{x^2}$]

$$\Rightarrow \quad v \frac{dv}{dx} = -\frac{g_0 R^2}{x^2}$$

$$\Rightarrow \quad v\, dv = -g_0 R^2 . x^{-2} dx$$

Integrating $\frac{v^2}{2} = -g_0 R^2 . \frac{x^{-1}}{-1} + c$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{g_0 R^2}{x} + c \quad \ldots(1)$$

where $c$ is a constant of integration